Lecture 23

Given a real hypersurface $M \subseteq \mathbb{C}^{n+1}$ (or more generally, a CR mfld of h-surface type), we have discovered several CR invariants that lead to different behavior (e.g. holom. extension of CR functions).

- Finite type, minimality.
- Levi form (ψcvxity, nondeg., eigenvalues)

Now, examine one important class of hypersurfaces in more detail.

Strictly ψcvx hypersurfaces.

Recall that a CR mfld M of CR dim $= n$, CR codim $= 1$ (h-surf.) is strictly ψcvx if the Levi form (at every $p \in M$) is definite (nondeg + all eigenvalues of same sign).

If $M \subseteq \mathbb{C}^{n+1} \Rightarrow \exists$ local coord's (near every $p \in M$)
$(z,w) \in \mathbb{C}^n \times \mathbb{C}$ s.t.

$$M: \quad \operatorname{Im} w = |z|^2 + O(|(z, \operatorname{Re} w)|^3)$$

$$|z_1|^2 + \dots + |z_n|^2$$

For s. ψcvx M, the CR invariants introduced all all the same. Need finer invariants to tell these apart.

<u>Ex.</u> Consider in $\mathbb{C}^2$

$$M_0: \quad \operatorname{Im} w = |z|^2$$
$$M_1: \quad \operatorname{Im} w = |z|^2 + \operatorname{Re}(z^2 \bar{z}^4)$$
$$M_2: \quad \operatorname{Im} w = |z|^2 + |z|^8$$

All s. ψcvx, but $\nexists$ bihollom. $H: (\mathbb{C}^2, 0) \circlearrowleft$ s.t. $H(M_i) = H(M_j)$ for $i \neq j$ (in $\{0,1,2\}$).

This can be checked in finite time by making an Ansatz $H = (f, g)$ and

$$z \to f(z,w), \quad w \to g(z,w)$$

but this is painful. The statement follows by Cartan–Chern–Moser theory.

- M_0 is <u>spherical</u> ($\Rightarrow$ <u>umbilical</u> at $(0,0)$).
- M_1 is <u>non-umbilical</u> but <u>obstruction flat</u> at $(0,0)$.
- M_1 is <u>umbilical</u> but <u>not</u> <u>obstruction flat</u> at $(0,0)$.

CCM Theory $\rightsquigarrow$ complete (countable) set of invariants for s. ψcvx h-surf.